Optimality of Imitative Behavior in Cournot Oligopoly

Possajennikov, A.

Publication date:
1999

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December 1998

Abstract

The paper considers a model of imitation in the context of Cournot oligopoly. Purely imitative behavior can lead to an outcome inconsistent with Nash equilibrium. The question is when we can reconcile imitation with the concept of Nash equilibrium. The paper extends purely imitative behavior in two ways. First, imperfect imitation is introduced. Second, a random matching and local interaction model is analyzed. Such variations in the imitative behavior improve efficiency and restore Nash equilibrium as the likely outcome of the dynamic imitation process.

Keywords: Imitation, Cournot oligopoly, evolutionary games
JEL Codes: C72, D43

* I thank Eric van Damme, Martin Dufwenberg, and Maria Montero for useful suggestions. The paper also benefited from comments of participants of Spring Meeting of Young Economists'98 and CentER seminar. Financial support of CentER for Economic Research is acknowledged.

1 Introduction

People often observe what their neighbors do. When planning to buy a computer, or a car, we often ask our friends or colleagues and follow their advice with respect to which brand is the best. It is often worthwhile to compare your own experience with somebody else's and do what the other person did if he/she fared better. Imitative behavior is part of the real life and we often employ it, maybe unconsciously.

In decision problems agents can observe what other individuals have done in the same situation as they are in. Clearly, observing a play of another person in the same situation gives some valuable information as you can learn via his/her mistakes instead by your own[1]. However, this extra information should be handled with care. If you imitate too quickly, you may end up in an inferior position[2]. An example is given in the literature about technology adoption, where population may end up in an inferior action if the agents disregard their own information in view of behavior of others[3]. The problem in decision problem in not whether to imitate but, rather, how to imitate.

In games players can imitate too. They want to improve their performance, therefore if they observe a payoff higher than their own, they might be tempted to imitate the strategy bringing higher payoff. Doing so may change their own payoff but at the moment of decision players do not realize it. One interpretation may be that players do not know that they are playing a game, therefore they use the same behavioral rule as in decision problems or use some other justification of the rule (like "other players might be more clever, I shall imitate their behavior", "finding really best strategy is too much work, let's watch what others do"). Imitation can be useful when environment changes as it is argued in Rhode and Stegeman (1997). There is experimental support for imitative behavior in games, including oligopoly models, nicely represented in Huck *et al.*(1998). Another interpretation of imitative behavior is that players are concerned about relative payoffs[4]. A non-strategic behavior in games, like imitation, may or may not bring the

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[1] Somebody said "fools learn on their own mistakes; wise people learn on mistakes of others"
[2] See e.g. Björnerstedt and Schlag(1996) and Schlag(1998).
[3] Examples in the literature are Ellison and Fudenberg (1993,1995).
[4] Such an interpretation appears e.g. in Palomino(1996).

outcome that would have appeared if strategic behavior were considered. This is one of the main questions of the present paper.

Specifically, in games interaction and imitation occur in the same population[5]. This case is of practical interest. For example, firms in an industry can be competing with each other and at the same time be interested in each other's market strategy so that a successful firm might be imitated by others. Another justification is that a firm with higher profit increases its market share or expands, still employing the same strategy, while other firms shrink, thus with time the strategy employed by the firm with higher profit gains more weight in the population. Of course, it does not mean that everybody will be better off at the end; on the contrary, the results might be very inefficient for the firms as in the Cournot oligopoly model of Vega-Redondo(1997).

The paper focuses on games arising from Cournot oligopoly models. Imitation and experimentation in oligopoly games were analyzed in Vega-Redondo(1997) and Gale and Rosenthal(1996), though interpretation of both components of the process differs in those two papers. The results of the papers also differ, Vega-Redondo got Walrasian (competitive) production level as the outcome while Gale and Rosenthal got Cournot-Nash equilibrium. The latter result was due to the fact that imitators were imitating the aggregate action of the population, disregarding comparison of payoffs on their current action and new action. We want to have imitation of success, in this respect our model follows closely the one of Vega-Redondo. However, then we are left with the problem that the outcome of the process may be inefficient for the firms. Therefore, we introduce variations of the imitation process or the underlying game that allow reinstalling "as if" rational behavior, represented by Nash equilibrium.

The first variation concerns imperfection in imitation. Imitation in real life can be imperfect, for example when one observes only a part of an action of another player but not the whole action. Then he/she attempts to imitate the action filling the absent part with something else, for instance, with part of his/her old strategy. In games such models were considered in the context of repeated prisoners' dilemma game by Ruebeck(1996) and Cooper(1996). Both substantial amount of coordination and total defection were

[5] The case where interaction and imitation are in different populations is considered in Schlag(1998).

obtained as outcomes depending on the exact specification of the model. In the present paper we extend one-stage Cournot oligopoly to two-stage capacity-price game and we demonstrate that it gives support to the Cournot-Nash outcome.

The second variation differentiates interaction and imitation in the sense that one can observe (and imitate) a player with whom he/she did not directly interact. For example, players interact in different markets but the information is spread to other markets too[6]. The possibility of observing another copy of the game allows comparing different outcomes of the game and restores the Nash equilibrium if it is more efficient than the outcome of purely imitative process. A similar model was considered in Palomino(1996), though there was no experimentation by the players.

The paper is constructed as follows. Section 2 presents the general model and an example of Cournot oligopoly game. Section 3 extends the example to imperfect imitation and Section 4 to a local interaction models. Section 5 concludes.

2 The model

The general case

There are $N = m \cdot n$ agents in the population, where m can be finite or infinite. They play an n-player symmetric game having been matched each period randomly or deterministically according to a prespecified rule; m is interpreted as the number of locations. Let S be the strategy set of the game, assumed finite. Agent i is characterized at time t by a pure strategy s_i^t he/she plays in period t.

The state of the population at time t is a vector $s^t \in S^N$, $s^t = (s_1^t, \ldots, s_N^t)$ of strategies of all agents. Let $\pi^t(s^t) = (\pi_1^t, \ldots, \pi_N^t)$ be the vector of realized payoffs of the players.

The imitation process works as follows. At each period of time each player receives with probability τ a possibility to revise his/her strategy, independently of other players. In what follows we take $\tau = 1$, since it simplifies the analysis considerably[7]. The

[6] Examples of applications of local interaction to evolutionary games are Ellison(1993) and Ely(1995).
[7] The results do not differ for $\tau < 1$.

player samples k other players randomly according to a prespecified sampling procedure and observes their payoffs and corresponding strategies at the previous period. We usually keep restrictions that players observe their own payoffs and strategies and those of their direct opponents in the n-player game. They can receive information about play in another copy of the n-player game, though. Denote $IP_i^t = ((\pi_{j1}^{t-1}, s_{j1}^{t-1}), \ldots, (\pi_{jk}^{t-1}, s_{jk}^{t-1}))$ the information the player has obtained and $P_i^t = (j_1, \ldots, j_k)$ the set of players information about whose payoffs and strategies player i has observed. The player finds the maximal payoff in his information set IP_i^t and then copies the corresponding strategy. That is, the player chooses s_j^{t-1} such that $j \in argmax_{k \in P_i^t} \{\pi_k\}$. If there are several strategies that give maximal payoff, one of them is chosen randomly according to a probability distribution with full support on the set of strategies giving highest profit[8]. The strategy of the player in period t is $s_i^t = s_j^{t-1}$. We call this kind of imitation "imitate the best". If a player did not receive a possibility to revise the strategy this period he/she keeps the old strategy, that is $s_i^t = s_i^{t-1}$.

If all individuals in the population play the same strategy, they all receive the same payoff since the game is symmetric. The imitation process alone cannot bring a new strategy in the population. Conversely, if there are several strategies in the population, even if they all bring the same payoff, there is non-zero probability that players will switch among them, settling eventually on the state where only one strategy is played. We can state

Observation 0 All states with strategy profile of the population $s^t = (s_1^t, \ldots, s_N^t)$ with $s_i^t = s_j^t \;\forall\, i,j$ are stationary states of the imitation process. These are the only stationary states of the process.

In order to analyze which stationary states are more stable we introduce the possibility of experimentation. In each period each player has independent probability λ to experiment, that is to switch to another strategy. The probability distribution over

[8] If we assume that if current payoff of a player is among highest, she does not change strategy, then stationary states other than monomorphic are possible, which makes the analysis more difficult but does not change qualitative results.

strategies resulting from experimentation will usually be taken uniform, unless specified otherwise. Then the combined process of imitation and experimentation defines an ergodic Markov chain on the space of states of the population, which has a stationary distribution $\mu(\lambda)$. We are interested in the case when the probability of experimentation is arbitrarily small, i.e. we consider $lim_{\lambda\to 0}\mu(\lambda)$.

Notice that we require very little rationality from the players. First, they do not suspect that they are playing a game: they simple copy others that may be their direct opponents! Second, they condition their behavior only on last period payoff, i.e. they have one-period memory. However, these boundaries on rationality of the players are not uncommon in the literature. For example, Björnerstedt and Schlag(1996) and Schlag(1998) analyzed imitative behavior in one-player decision problems, where players are allowed to observe only one other individual in one period. The closest to our model are the model of Vega-Redondo(1997) where players could observe all other players in the population in the setup of Cournot oligopoly and the model of Palomino(1996) where players could observe only a subpopulation of other players. The aim of the present paper is to show that even with little rationality the players can produce “as if” rational behavior.

A simple example of the process described above is a population of 2 agents, playing 2-player game (thus $m = 1$, $n = 2$). In this case the outcome of the process depends on relative, and not on absolute payoffs[9]. How exactly it depends for general games is a question for future research, though in Section 3 we present an example showing that they do not coincide. In this paper we consider symmetric games with a pure strategy symmetric equilibrium and the main question is to what state the imitation and experimentation dynamics converges.

An example

[9] Shubik(1982, Section 10.3) was one of the first to notice this. See also Schaffer(1989).

One of the results in the class of imitation models described above is due to Vega-Redondo(1997), who shows that a non-Nash Walrasian outcome is achieved in Cournot oligopoly. We shall build on this model.

Consider twice differentiable decreasing inverse demand function $P(Q)$ with $P''(Q)q_i < -2P'(Q)\ \ \forall q_i, Q$. The cost function $C(q)$ is assumed to be convex, $C(0) = 0$. To bring the setup under the general model considered above, assume that firms can choose quantities from a finite grid $\Gamma_q = \{0, \delta, \ldots, v\delta\}$. The grid is assumed to be fine enough to contain the necessary quantities like Walrasian output and Cournot output[10].

We can illustrate the work of the imitation process on the example with linear cost function. Notice that if price is larger than marginal cost the profit of one firm is higher than that of the other if it has higher output since the difference between price and marginal cost is the same for all firms. If price is below marginal cost, all firms make losses but the one with lower output makes a lower loss. If firms are already in a steady state, i.e. they produce the same quantity and if price is higher than the marginal cost, then a firm can experiment with a higher quantity (still keeping price above marginal cost) and obtain a higher profit than the other firm. The other firm then imitates new quantity and the old equilibrium is upset. If current price is lower than marginal cost, a firm can experiment with lower quantity, thus having lower loss than the other firm does. Finally, if price equal marginal cost no experimentation by one firm can upset this state.

Therefore whatever the current output is, a firm will get more than its opponent if it increases or decreases output till point where price equal marginal cost. If both produce competitive output such as the price equal marginal cost, experimentation will not bring to a firm profit higher than the profit of its competitor, while at all other production levels experimentation upsets corresponding steady states. The process then goes to the competitive output level. The result holds for arbitrary convex cost function.

Proposition 1 (Vega-Redondo(1997)) The long-run outcome of imitation and experimentation dynamics for Cournot oligopoly is the Walrasian equilibrium.

The outcome of the process for Cournot oligopoly coincides with Walrasian outcome and the latter is Nash equilibrium of the game where relative profit is maximized. However, it is not always the case that the result of the process coincides with relative profit maximization, as we shall demonstrate in an example in Section 3.

The result of imitation and experimentation dynamics in the simple Cournot oligopoly is not efficient for the firms. If they simply imitate each other and occasionally experiment they receive less profit then in Nash equilibrium. This result also holds when a higher payoff is not always imitated. Schlag (1998) argued that proportional imitation, that is when the probability of imitation is proportional to the difference in payoffs, is "optimal"[11] for a situation when agents play many copies of a decision problem. This slows down the process but does not change the long run outcome in our case since a strategy close to Walrasian equilibrium gets imitated with higher probability.

Despite the inefficiency in the outcome, imitation does not seem to be a particularly unplausible process and people do imitate in real life and experiments (see Huck *et al.*(1998)). Is it possible to change the setting in the model slightly such that they will fare better? Next two sections build on this result and extend the model in several dimensions.

3 Imperfect Imitation

An imperfection in imitation can occur when players can observe only realized play but not the intended strategy like in extensive form games. Imitation in the context of repeated prisoner's dilemma was considered in Ruebeck(1996) and Cooper(1996). The players there could observe only realized play and therefore they were not able to distinguish between “always cooperate” and “tit-for-tat” if they never deviated themselves. To incorporate such possibility into oligopoly model, we extend the game to

[10] The result can be extended to continuum strategy set, see Schenk-Hoppé(1997). We assume finite set to avoid unnecessary technical difficulties.
[11] That is, payoff increasing in the next round for any decision problem.

two-stage game of price competition with capacity precommitments, the model considered in Kreps and Scheinkman(1983)[12].

They consider a game where in the first stage two firms choose capacities and in the second stage choose prices. The assumptions on the demand and cost functions are the same as in Section 2. The rationing rule is efficient, i.e. consumers with higher valuations buy from the cheapest supplier. Kreps and Scheinkman then show that under certain mild conditions the choice of capacities and prices corresponding to Cournot outcome of one-stage game is an equilibrium of the two-stage game.

Under imperfect imitation a firm can observe only price announced by the other firm after they both have chosen capacities, and it does not know what the other firm strategy is in response to other capacities. Therefore a firm can imitate just one price and one capacity. There might be different ways to model such a restriction. For example, the firms can keep their intended responses on capacity combinations, different from the observed one, intact. Or they can change all intended price choices to the one observed. We adopt the second way of modeling, though it seems to be quite a restriction on rationality of the players. However, it is the way that requires least memory to remember a strategy (only one price to remember). We also believe that other possible ways to model price imitation would not change the qualitative result.

The restriction we pose on the imitation of prices leads to the situation where firms announce both capacity and price simultaneously, thus reducing the original two-stage game to a one-stage game. This has as a result that there are multiple Nash equilibria: choosing Walrasian quantity and price is an equilibrium as well as choosing Cournot quantity and price. This makes it more interesting that Cournot quantity and price have non-zero probability to be the long-run outcome. To make the strategy set finite, assume that, in addition to the grid on quantities, the firms choose prices from a finite grid $\Gamma_p = \{0, \delta, \ldots, w\delta\}$.

Let us illustrate the work of the process on a simple example with linear demand and cost functions and only two possible capacities and two possible prices,

[12] Another game of this sort, related to Bertrand differentiated product game, is Hotelling game with quadratic transportation cost. The equilibrium is to be on extremes but if a firm moves to the middle of the interval, it hurts the other firm more than itself.

corresponding to Cournot-Nash and to Walrasian competitive outcomes. Then each firm has 4 strategies: $(q_N,p_N),(q_N,p_C),(q_C,p_N),(q_C,p_C)$ where q_N (p_N) stands for Cournot-Nash capacity (price) and q_C (p_C) stands for competitive capacity (price). The demand and cost functions are $P(Q) = a - bQ$ and $C(q) = cq$, with positive a,b,c and $a > c$. Then the Cournot-Nash output of each firm is $q_N = (a - c)/3b$, corresponding price is $p_N = (a + 2c)/3$, competitive output of each firm is $q_C = (a - c)/2b$, corresponding price is $p_C = c$. After dividing each payoff by $(a - c)/18b$, which does not change the structure the game has the following form:

	(q_N,p_N)	(q_N,p_C)	(q_C,p_N)	(q_C,p_C)
(q_N,p_N)	$2a-2c$, $2a-2c$	$2a-2c$, 0	$2a-2c$, $2a-5c$	$a-4c$, 0
(q_N,p_C)	0 , $2a-2c$	0 , 0	0 , $2a-5c$	0 , 0
(q_C,p_N)	$2a-5c$, $2a-2c$	$2a-5c$, 0	$2a-5c$, $2a-5c$	$a-7c$, 0
(q_C,p_C)	0 , $a-4c$	0 , 0	0 , $a-7c$	0 , 0

Game 1

The imitation process has 4 steady states on the diagonal of the bimatrix. If $a > 4c$, the steady state corresponding to the Cournot-Nash outcome equilibrium cannot be upset by experimentation of one firm, while other steady states can be upset. Therefore, in this case the Cournot-Nash outcome is obviously played in the long run.

If $a < 4c$, both Cournot (q_N,p_N) and competitive (q_C,p_C) outcomes are Nash equilibria of the normal form game. Experimentation by one firm is enough to upset any of the steady states, for example (q_N,p_N) is upset by one player moving to (q_C,p_C). (q_C,p_C) itself is upset by one firm experimenting with (q_N,p_C). What we need to show is that the Cournot-Nash outcome can be achieved from other steady states with no more experimentations than any other steady state. Using graph-theoretic techniques introduced to evolutionary game theory by Kandori *et al.*(1993) and Young(1993) (more on them in the Appendix), we can show that Cournot-Nash outcome has non-zero probability in the limit stationary distribution[13]. In general case, similar reasoning gives following

Proposition 2 In the model of imitation described above the Cournot-Nash outcome has non-zero probability in the limit stationary distribution.

The proof is in the Appendix.

Notice, however, that the Walrasian outcome still has strong positions as the long-run outcome. It has non-zero probability in the limit distribution too, since, unless we are in the Walrasian equilibrium, where marginal benefit equal marginal cost, it is always possible to undercut slightly the other firm in price and expand the capacity such that the deviating firm profit does not change but the remaining firm demand decreases thus decreasing its profit. The remaining firm then is forced to follow the deviation.

The example presented here gives also answer on the question whether imitation and experimentation dynamics leads to the outcomes which are Nash equilibria of a game where absolute payoffs are replaced by relative payoff. Consider the example above with $a = 2, b = c = 1$. The bimatrix of relative payoffs is

	(q_N,p_N)	(q_N,p_C)	(q_C,p_N)	(q_C,p_C)
(q_N,p_N)	*0 , 0*	*2 , -2*	*3 , -3*	*-2 , 2*
(q_N,p_C)	*-2 , 2*	*0 , 0*	*1 , -1*	*0 , 0*
(q_C,p_N)	*-3 , 3*	*-1 , 1*	*0 , 0*	*-5 , 5*
(q_C,p_C)	*2 , -2*	*0 , 0*	*5 , -5*	*0 , 0*

Game 2

Let us denote p_1,p_2,p_3,p_4 (q_1,q_2,q_3,q_4) the probabilities with which strategies are played by Player 1 (Player 2). The Nash equilibria of this game are given by $p_1 = q_1 = 0,$

[13] For Game 1 with $a < 4c$, the limit stationary distribution puts weights (¼,¼,0,½) to steady states $((q_N,p_N),(q_N,p_C),(q_C,p_N),(q_N,p_N))$ respectively.

$p_2 \leq 1/2$, $q_2 \leq 1/2$, $p_3 = q_3 = 0$, $p_4 = 1 - p_2$, $q_4 = 1 - q_2$. The strategy (q_N, p_N) is never played in equilibrium while the limit distribution of the imitation and experimentation dynamics places non-zero weight on it. Therefore, in this game the outcome of imitation and experimentation dynamics is a larger set than the set of Nash equilibria of a game where absolute payoffs are replaced by relative payoffs. Nevertheless, they seem to be related and the investigation of it is left for future research.

This section showed that imperfect imitation may restore the outcome corresponding to Nash equilibrium, where the firms earn higher profits than in the competitive outcome. Possible variations on this might be imperfect imitation in normal form games where a player can imitate only subset of the other player strategies, or a player can experiment only locally. These possibilities are left for future research.

4 Random matching and local interaction

In this section we consider models with $m > 1$. Specifically, we will assume there is a large (infinite) number of individuals in the population. They are randomly matched to play an n-player Cournot oligopoly game.

Let us consider first the process without mutations. We focus on the analysis when there are only two strategies present in the population. This is not a strong restriction though since with only imitation the number of strategies in the population cannot increase, but can decrease till there are only two left. We call the situation with only two strategies a two-strategy contest. If we find strategy s which wins every two-strategy contest without experimentation, then we can say that such a strategy is a long-run outcome of imitation and experimentation process since any other monomorphic state of the population can be upset by experimentation with s. The state where everybody plays s cannot be upset by unilateral experimenting with other strategies.

Suppose only Cournot-Nash and Walrasian strategies are present in the population. Denote by $\pi_N(k)$ $(\pi_C(k))$ payoff for player playing Cournot-Nash (Walrasian) strategy while k players the player is matched with play Cournot-Nash strategy (and thus $n - k$ players play Walrasian strategy). π's are increasing in k, $\pi_N(k) < \pi_C(k)$ $\forall k < n$ and assume $\pi_N(n) > \pi_C(k)\ \forall k < n$. Assume the worst possible case for Cournot-Nash strategy, $\pi_N(n - 1) < \pi_C(1)$. These assumptions represent general properties of Cournot oligopoly, namely that the deviation from Cournot-Nash equilibrium makes a firm worse off, but hurts its competitors more.

When updating their strategy, players sample $n + k$ other players, among which n their direct opponents in the match and k other individuals. Sampling of k others is uniform. Then they copy strategy that gives the highest payoff among sampled.

First consider the case $k = 1$. Let α_t be the proportion of the population playing Cournot-Nash strategy at time t. If such a player is matched with other players playing Cournot-Nash strategy, she gets the highest possible payoff and therefore will not change strategy. If this player is matched with some players who play Walrasian strategy, she will switch to their strategy unless she samples a player getting highest possible payoff, which occurs with probability α_t^n. Correspondingly, a player playing Walrasian strategy will not switch to Cournot-Nash strategy unless she samples the same player. The population imitation dynamics can then be described as

$$\alpha_{t+1} = \alpha_t - \alpha_t(1-\alpha_t^n)(1-\alpha_t^n) + (1-\alpha_t)\alpha_t^n$$

This equation has steady states 0, 1, and $\alpha^* \in (0,1)$. α^* is unstable steady state, thus both 0 and 1 have non-empty basins of attraction and both can be selected by a process with experimentation.

As k increases, the probability of sampling a player with highest possible payoff increases, approaching 1 as k goes to infinity. The formula for population dynamics becomes

$$\alpha_{t+1} = \alpha_t - \alpha_t(1-\alpha_t^n)(1-\alpha_t^n)^k + (1-\alpha_t)(1-(1-\alpha_t^n)^k)$$

For any $\alpha_t > 0$ it is possible to find k such that $\alpha_{t+1} > \alpha_t$. Then the dynamics converges to the state $\alpha_t = 1$, that is everybody plays Cournot-Nash strategy. If we introduce experimentation, small number of experimentators in the state $\alpha_t = 0$ would be enough to upset the state.

Notice that above construction works also for other strategies. If we compare, say, joint profit maximizing strategy to Cournot-Nash strategy, a firm deviating from join profit maximum while other firms stay has higher profit than it had in the maximum. Therefore, more firms will follow it, so the joint profit maximum is not stable. Thus, we have

Proposition 3 In the model with infinite population playing Cournot oligopoly game above with random matching, the Cournot-Nash strategy wins every two-strategy contest if sample size k is large enough.

Corollary The Cournot-Nash strategy is the long-run outcome of the imitation and experimentation process if sample size k is large enough.

Palomino(1996) also considered similar games and showed that the strictly dominated Walrasian strategy survives if players have information about less than the whole population. Our model differs from his in two respects. First, in a sense we change the order of limits taken: we first let k grow keeping initial proportion α_0 fixed. In Palomino(1996) it was shown that for any finite k there exists α_0 that the process converges to the strictly dominated strategy. Second, by introducing experimentation we analyze stochastic stability of the stationary states of the imitation dynamics, one of which is always the state where all players play a strictly dominated Walrasian strategy.

For finite population one can achieve a positive result for local interaction model. Suppose the locations of the firms are fixed. The firms in a location interact among themselves but not with "outside world". However, firms also have information about firms in some other location. If there is a location playing Nash equilibrium, the firms in the location will never change their strategies. Firms in the other pairs sooner or later will sample a firm in the pair and change their strategy to Cournot-Nash one. Therefore the state when the whole population plays Walrasian strategy can be upset by n

experimenting firms if they happen to be in the same location, while for the state where everybody plays Cournot-Nash strategy it is not enough. The Nash equilibrium is the unique long run outcome.

Observation 1 In finite population with local interactions playing Cournot oligopoly game above the Nash equilibrium is the unique long run outcome for any $k > 0$.

In context of industrial organization the model can be interpreted as if there are several similar industries which having a number of firms, or several geographical or otherwise markets. In random matching model firms are randomly assigned to markets while in local interaction they their locations are fixed. The firms can observe their own industry or market statistics and they can also sample firms in other industries or markets. Interesting extensions might be when firms have information about average performance of strategies in the population and when neighborhoods are overlapping, e.g. geographically or in product space.

This result is similar in spirit to the result by Ely(1995). In his model players can choose locations and they prefer the most efficient one. In our model, player can observe other locations and, again, if they observe a more efficient one, they will imitate it. Notice that our model will not work for prisoner's dilemma: even if a pair of cooperators will convert some other player to cooperation, a defector will exploit that cooperator which might induce original cooperators to switch to defect too.

The results presented in this section showed that while imitating it is good to have some external information, not only the information about your direct opponents. The intuition for the result is simple and resembles the intuition for imitation in decision problems. If one never observed an efficient way to solve a problem, one could not come to it through imitation. However, if an efficient solution exists somewhere, it will be found.

5 Conclusion

We have considered several models of imitation in oligopoly games. Though the results cast doubts on (perfect) imitation, the imitative behavior is not as unattractive as it may seem: taken with care, it produces plausible results. Actually, one of the most celebrated strategy in repeated prisoner's dilemma, “Tit-for-tat” is nothing else than imitation of what the other player did in the previous period. Applied with care, that is start from cooperative strategy, “Tit-for-tat” “solves” the dilemma. Therefore, one cannot reject imitative behavior without considering the game at hand.

The model of imitation as presented here is applicable only to symmetric games. Still, asymmetric games might be considered. For example, in sequential extensive form, it is possible to imitate preceding player move. In chess, for example, there was a (wrong) belief that if Black imitates White, it can achieve a draw. Of course, it is not true, but there might be games where it is! Another possibility, more applicable to industrial organization, is when one firm does not know cost function of the other, or it does not know market parameters, and, therefore, imitate the other, more "mature" firm. Another line of future research tries to link more closely imitation and relative profit maximization and then analyze the evolutionary stability of relative profit maximization.

Appendix

The proofs of the Proposition 2 will be based on two lemmas.

Lemma 1

$\forall(q,p)$ with $q \neq D(p)/2$ one mutation is needed to arrive at $(D(p)/2,p)$ if marginal revenue at (q,p) is higher than marginal cost.

Proof

Consider $q < D(p)/2$, that is the firms undersupply the market. Therefore, if a firm increases q to $D(p)/2$, it sells more and has higher profit since marginal revenue is higher than marginal cost. The other firm then imitates. If $q > D(p)/2$, the firms oversupply the market, that is they both sell only $D(p)/2$. Thus, if a firm decreases q till $D(p)/2$, it has lower costs and higher profit. The other firm follows.

Lemma 2

$\forall(q,p)$ with $q \neq D(p)/2$ there is a path through steady states to $(q,D^{-1}(2q))$, each step requiring one mutation, if profits are positive along the path.

Proof

Consider $q < D(p)/2$. If one firm increases price to $D^{-1}(2q)$, its demand is still q as well as the demand of the other firm. Therefore, the revenue of the deviating firm is higher, while the cost is the same. The deviating firm has higher profit. The other firm imitates. If $q > D(p)/2$, the firms sell $D(p)/2$. If one firm reduces price marginally it sells q now, while the other firm sells less. Therefore deviating firm has higher profit (since it was assumed that profits were positive) and the other firm imitates until $p = D^{-1}(2q)$.

Proposition 2

In the model of imitation described in Section 3 the Cournot-Nash outcome has non-zero probability in the limit stationary distribution.

Proof

We apply the technique of z-trees, used in evolutionary games by Kandori *et al.* (1993) and Young(1993) (see also Vega-Redondo(1997)). Let Z be the set of all states. For a given state z, a z-tree is a directed graph with states as vertices, such that

(i) each $z' \neq z$ is the source of exactly one arc;

(ii) from every $z' \neq z$ there is a path from z' to z.

Since we are concerned with the process when the probability of mutation becomes arbitrarily small, we are interested in probabilities of transition from state to state in terms of λ. These probabilities are polynomials in λ. Therefore, only the order of λ, or the number of mutations needed to go from one state to the other, matters. A cost is assigned to each arc representing the minimal number of mutations (experimentators) needed for transition from one state to the other. The cost of a z-tree is the sum of the costs of the arcs in the tree. The minimal z-tree for given z is one with minimal cost among all z-trees for it. The minimal tree is one with minimal cost among all z. Then the weight of a state in the limiting stationary distribution is equal to the number of minimal z-trees for it which cost equal to the cost of the minimal tree divided by the total number of minimal trees for all states.

The imitation process without mutation has $v \cdot w$ steady states (recall that v is the cardinality of the capacity space grid, w is the cardinality of the price space grid). From any other state the imitation process leads to one of the steady states without mutation (i.e. with cost 0), so we can focus on steady states of the process.

Since there are $v \cdot w$ steady states, the minimal z-tree for any of them has cost $v \cdot w - 1$. We shall construct a tree with such cost for the state (q_N, p_N). First, consider arbitrary (q,p) with $q \neq D(p)/2$. If $q < D(p)/2$, by Lemma 2 we can achieve the curve $q = D(p)/2$ by increasing price by one mutation. If $q > D(p)/2$, by Lemma 1 we can arrive at the curve $q = D(p)/2$ by decreasing capacity in one mutation.

Now consider arbitrary (q,p) with $q = D(p)/2$. Consider $q < q_N$ which implies $p > p_N$. Recall that reaction functions in Cournot oligopoly are downward sloping implying that if $q < q_N$, then for q' such that $q < q' < q_N$ and appropriate price the profit is higher than if a firm chooses (q,p). Therefore, in one mutation we move to a state with q closer to q_N after which again in one mutation we move to (q,p) with $q = D(p)/2$. If $q > q_N$, then for q' such that $q_N < q' < q$ and appropriate price profit is higher. Similar reasoning as above applies.

We have shown that for any steady state there is a path through other steady states leading to Cournot-Nash outcome with a cost of each step equal to 1. Therefore, the minimal z-tree for Cournot-Nash outcome has cost of $v \cdot w$, not more than the minimal tree for all states.

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